Published by Institute of Physics Publishing for SISSA

Received: *January 25, 2005*
Revised: *April 25, 2005*
Accepted: *May 26, 2005*
Published: *June 16, 2005*

# Branes in the plane wave background with gauge field condensates

**Tako Mattik**

*Institut für Theoretische Physik, ETH Zürich*
*CH-8093 Zürich, Switzerland*
*E-mail:* mattik@itp.phys.ethz.ch

Abstract: Supersymmetric branes in the plane wave background with additional constant magnetic fields are studied from the world-sheet point of view. It is found that in contradistinction to flat space, boundary condensates on some maximally supersymmetric branes necessarily break at least some supersymmetries. The maximally supersymmetric cases with condensates are shown to be in one to one correspondence with the previously classified class II branes.

Keywords: Penrose limit and pp-wave background, D-branes.

JHEP06(2005)041

## Contents

## 1. Introduction

Since the discovery of the plane wave in [1] as another maximally supersymmetric background of type-IIB superstring theory and the explicit quantization of strings on it in [2, 3], this solution has been intensively used in the study of the gauge-gravity (AdS/CFT) correspondence via the BMN proposal [4]. For reviews see for example [5 – 9]. Branes in this Ramond-Ramond background have been studied in a number of papers from different points of view. The probe brane approach was carried out in [10 – 12], boundary states were used in [13 – 16] and open string theory methods were applied in [17 – 21]. Closely related setups were considered for example in [22 – 26].

In this paper we study branes in the plane wave with nonzero gauge field condensates $\mathcal{F}^{IJ}, \mathcal{F}^{-I}$ from the world-sheet point of view. We derive the conserved (dynamical) supersymmetries, calculate open string partition functions and prove the equivalence with the closed string cylinder diagrams, expressed by boundary state overlaps.

It will be shown that in contradistinction to the situation in flat Minkowski space it is impossible to turn on magnetic fields on some supersymmetric branes in the plane wave without further reducing the amount of conserved supersymmetries.

Maximally supersymmetric branes in the plane wave without boundary condensates were classified in [18, 15]. Here we will follow the conventions of [14, 15] in which in particular the plane wave metric is given by

$$ds^2 = 2dX^+dX^- - \mu^2 X^I X^I dX^+dX^+ + dX^I dX^I \tag{1.1}$$

and the nonzero components of the $R \otimes R$ five-form field strength by

$$F_{+1234} = F_{+5678} = 2\mu\,. \tag{1.2}$$

The classification of supersymmetric branes in [18, 15] proceeds by using the fermionic gluing matrix $M$ which is given as in flat space by the product of $\gamma$-matrices along the Neumann directions

$$M = \prod_{I\in\mathcal{N}} \gamma^I\,. \tag{1.3}$$

The class I ($D_-$) branes are characterised by $M\Pi M\Pi = -1$ with $\Pi = \gamma^1\gamma^2\gamma^3\gamma^4$, and are of the form $(r, r+2)$, $(r+2, r)$ with $r = 0, 1, 2$. Here the notation $(r, s)$ labels the orientation of the branes with respect to the SO(4) $\times$ SO(4) - background symmetry.

The so called class II ($D_+$) branes are characterised by $M\Pi M\Pi = +1$, and consist of the (0,0) instanton and the (4,0), (0,4) branes [18, 15]. The last example is special, as these branes couple to the nonzero background $F_5$-form flux, leading necessarily to a nonzero $\mathcal{F}^{-I}$-worldvolume flux.

The gauge field condensates considered in this paper will be seen to give rise to new continuous families of maximally supersymmetric D-branes in the plane wave. These families interpolate smoothly between the euclidean (class I) (2,0), (0,2) branes and the (0,0) instanton and between the (4,2), (2,4) and the class II (4,0), (0,4) branes. Because of this, the new branes are in one to one correspondence with the previously mentioned class II branes.



JHEP06(2005)041

On the remaining static euclidean maximally supersymmetric (non-oblique) class I branes, the (3,1), (1,3) branes, constant magnetic fields reduce the supersymmetry further. This behavior will be seen to be related to the observation from [13, 17] that the (static) class I branes preserve the maximal amount of (dynamical) supersymmetries only when placed at the origin of transverse space.

As a consistency check we consider the equivalence of the open and closed string description of the branes under consideration. For this we extend and connect the results of [14, 15] by defining gauge field dependent generalizations of the special functions $f_i^{(m)}$ of [14] and $g_i^{(m)}$ of [15]. These special functions itself were $m$-dependent deformations of the well known $f$-functions of Polchinski and Cai from [27], which appear in the usual flat space treatment. Similar deformations of the standard $\Gamma$-function have appeared in the context of plane-wave physics in [28, 29].

The new function $g_2^{(m)}(q,\theta)$, for example, which appears in the cylinder diagram of (2,0) branes, interpolates between the $f_2^{(m)}(q)$ function from [14] and the $g_2^{(m)}(q)$ function from the instanton description of [15]. The angle variable $\theta \in [0,\pi]$ parameterizes here the world-volume gauge field strength $\mathcal{F}$.

Concerning the open/closed duality a certain subtlety derives from the fact that the standard light-cone gauge

$$X^+ = 2\pi P^+ \tau \tag{1.4}$$

leads in the closed string sector to boundary states of D(p+1) instantons, whereas in the open sector it gives rise to a description of Dp-branes, that is, branes wrapping both light-cone directions. Starting with boundary states one therefore has to apply a modified light-cone gauge condition in the open string sector with $X^+$ being proportional to $\sigma$ instead of $\tau$ to establish the open/closed duality. In this paper we follow the exposition in [14, 15] (see also [11] for a further discussion).

The situation of gauge condensates extended along a light-cone direction, that is, only $\mathcal{F}^{-I} = \mathcal{F}_{+I} \neq 0$, was addressed previously in [10, 15, 30] and for the more general massive backgrounds of [31] in [32]. Aspects of non-commutativity in the plane wave resulting from a nonzero $B$-field background were studied in [21, 33]. For other studies of D3 branes with condensates in this background by supercoset or DBI methods, see [34] and [35]. Additional comments on the related situation of branes in the Nappi-Witten background appeared recently in [36].

The paper is organized as follows. After briefly summarizing general aspects of boundary condensates on D-branes and a short review of the situation in flat space in section 2, we will give a closed string boundary state description of maximally supersymmetric branes in the plane wave by considering their (dynamical) supersymmetries in section 3. By doing so, we will derive a general condition for the gluing matrices of maximally supersymmetric branes, whose solutions will be discussed in section 4. In the following section 5, certain boundary states and their overlaps will be derived which will finally be compared with the results of a detailed open string treatment from section 6. Certain technical calculations are deferred to the appendix.



JHEP06(2005)041

## 2. Boundary condensates on Dp branes

Let us begin by reviewing some well-known facts about boundary condensates on D-branes. As described for example in [37 – 40] and references therein, the introduction of a boundary condensate with (constant) gauge potential $A$ on the world volume of a D-brane leads to the following boundary action

$$\int ds \left( A_I \partial_s X^I - \frac{1}{2} F_{IJ} S \gamma^{IJ} S \right) \tag{2.1}$$

with the (abelian) field strength $F = dA$. In the case of a constant $B$-field, the bosonic bulk term proportional to

$$\epsilon^{\alpha\beta} B_{rs} \partial_\alpha X^r \partial_\beta X^s \tag{2.2}$$

and the corresponding fermionic couplings become total derivative terms, and we obtain for a constant gauge field for which we can set $A_I = -\frac{1}{2} F_{IJ} X^J$ the combined surface action

$$\int ds \mathcal{F}_{IJ} \left( X^{[I} \partial_s X^{J]} - S \gamma^{IJ} S \right). \tag{2.3}$$

Here we have used the usual gauge invariant quantity $\mathcal{F} = F - B$.

For the Neumann directions this boundary action leads to the modified boundary conditions

$$\partial_\sigma X^I + \mathcal{F}_\nu^{IJ} \partial_\tau X^J = 0 \tag{2.4}$$

at $\sigma = 0, \pi$. By $\nu = 1, 2$ possibly different condensates on the branes at $\sigma = 0$ and $\sigma = \pi$ are distinguished. For simplicity we will, however, concentrate in the following on the case $\mathcal{F}_1 = \mathcal{F}_2$. As a relation between $\partial_+ X$ and $\partial_- X$ equation (2.4) reads

$$\left( \partial_+ X^I + N^{IJ} \partial_- X^J \right) = 0 \tag{2.5}$$

with

$$N^{IJ} = - \left[ \frac{1 - \mathcal{F}}{1 + \mathcal{F}} \right]^{IJ}, \qquad \mathcal{F}^{IJ} = \left[ \frac{1 + N}{1 - N} \right]^{IJ}, \tag{2.6}$$

which is valid for the Neumann directions. For the Dirichlet directions the condition (2.5) will be imposed with $N = 1$, that is

$$\left( \partial_+ X^i + \partial_- X^i \right) = \partial_\tau X^i = 0 \tag{2.7}$$

on the boundary.[1]

In [38] it is shown how to implement a nonzero gauge field condensate in the light-cone gauge boundary state description of branes in type-II string theory in a flat background by using the conservation of space-time supersymmetries as a guiding principle. Using boundary states this conservation is expressed by

$$\left( Q_{\dot{a}} + i\eta M_{\dot{a}\dot{b}} \tilde{Q}_{\dot{b}} \right) ||B\rangle\rangle = 0\,, \qquad \left( Q_a + i\eta M_{ab} \tilde{Q}_b \right) ||B\rangle\rangle = 0 \tag{2.8}$$

for the 16 dynamical and 16 kinematical supersymmetries preserved by the (flat) background.

[1] In this paper we will use upper case letters $I, J \ldots$ for Neumann — and lower case letters $i, j \ldots$ for Dirichlet directions.

As shown in [38], (2.8) is fulfilled when using the fermionic gluing conditions

$$\left(S_n^a + i\eta M_{ab}\widetilde{S}^b_{-n}\right)||B\rangle\rangle = 0 \tag{2.9}$$

in addition to the bosonic relations (2.7) and (2.5) described above, iff the orthogonal matrices $N^{IJ}, M_{ab}, M_{\dot{a}\dot{b}}$ appearing in these conditions are related by

$$M_{ab}\gamma^I_{b\dot{c}}M^t_{\dot{c}\dot{d}} = \gamma^J_{a\dot{d}}N^{JI}\,. \tag{2.10}$$

This equation expresses, as will be further explained later on, a SO(8)-triality relation.

As there are no further conditions on the gluing matrices, this means that an arbitrary constant boundary condensate can be turned on on any even dimensional world-volume subspace of a supersymmetric brane in flat space without changing the amount of conserved supersymmetries.

For further details and in particular the derivation of cylinder diagrams as boundary state overlaps, the reader is referred to [38, 39], for example.

## 3. Gluing conditions for maximally supersymmetric branes in the plane wave

Now we turn to branes in the plane wave background, being mainly interested in static maximally supersymmetric configurations. In the following we use the same conventions as in [15].

In terms of boundary states, the conservation of dynamical supersymmetries is as in flat space encoded in

$$\left(Q_{\dot{a}} + i\eta M_{\dot{a}\dot{b}}\tilde{Q}_{\dot{b}}\right)||B\rangle\rangle = 0\,, \tag{3.1}$$

where for consistency with the supersymmetry algebra $M$ has to be an orthogonal matrix. Assuming the usual Dirichlet conditions (2.7) for a flat D-brane which in terms of (closed string) modes reads

$$(a^i_n - \tilde{a}^i_{-n})||B\rangle\rangle = 0\,, \tag{3.2}$$

the condition (3.1) uniquely determines the fermionic gluing conditions. Using the mode expansion of the dynamical supersymmetries given in appendix A of [15], they can be determined to be of the form

$$\left(S^a_n + i\eta K^{ab}_n\tilde{S}^b_{-n}\right)||B\rangle\rangle = 0 \qquad (n \neq 0) \tag{3.3}$$

with

$$K_n = \frac{1 - \frac{\eta m}{2\omega_n c_n^2}\Pi M^t}{1 + \frac{\eta m}{2\omega_n c_n^2}M\Pi}M = \frac{1 - \eta\frac{\omega_n - n}{m}\Pi M^t}{1 + \eta\frac{\omega_n - n}{m}M\Pi}M\,, \tag{3.4}$$

where we furthermore used

$$\left[M, \gamma^i\right] = 0 \tag{3.5}$$

for the Dirichlet directions. The last equation will be further discussed in the open string setting later on. The formula (3.4) appeared already in [16] in the context of the oblique OD3 brane.

In a next step we have to test whether the contributions from the Neumann directions in (3.1) vanish when appropriate bosonic gluing conditions are enforced. Using (3.3) together with the relation

$$\left(a_n^I - N_n^{IJ}\tilde{a}_{-n}^J\right)||B\rangle\rangle = 0 \tag{3.6}$$

with a mode depending gluing matrix $N_n$ to be determined below, the boundary condition (3.1) leads to

$$0 = \sum_{n\in\mathbb{Z}}\left[\left(\gamma^I + \frac{m\eta}{2\omega_n c_n^2}M\gamma^I\Pi\right)a_{-n}^I S_n + i\eta\left(M\gamma^J - \frac{m\eta}{2\omega_n c_n^2}\gamma^J\Pi\right)\tilde{a}_n^J\tilde{S}_{-n}\right]||B\rangle\rangle \tag{3.7}$$

which simplifies to

$$\left(\gamma^I + \frac{m\eta}{2\omega_n c_n^2}M\gamma^I\Pi\right)N_{-n}^{IJ}K_n - \left(M\gamma^J - \frac{m\eta}{2\omega_n c_n^2}\gamma^J\Pi\right) = 0\,. \tag{3.8}$$

Using (3.4) and

$$\frac{m\eta}{2\omega_n c_n^2} = \eta\frac{\omega_n - n}{m} \tag{3.9}$$

this finally gives

$$\begin{aligned} &M\gamma^J M^t - \eta\frac{\omega_n - n}{m}\left(\gamma^J\Pi M^t - M\gamma^J\Pi\right) - \frac{(\omega_n - n)^2}{m^2}\gamma^J \\ &= N_{-n}^{IJ}\left[\gamma^I - \eta\frac{\omega_n - n}{m}\left(\gamma^I\Pi M^t - M\gamma^I\Pi\right) - \frac{(\omega_n - n)^2}{m^2}M\gamma^I M^t\right]. \end{aligned} \tag{3.10}$$

Assuming an $n$-independent gluing matrix $N_n = N$ as for example in [13], this would lead to the conditions

$$M\gamma^J M^t = N^{IJ}\gamma^I\,; \qquad \gamma^J = N^{IJ}M\gamma^I M^t \tag{3.11}$$

and therefore especially to $N^2 = 1$. As for consistency of the bosonic gluing conditions $N$ furthermore has to be orthogonal, this gives $N \equiv -1$ for the Neumann directions and thus we would be left with the situation of a vanishing boundary condensate. As in the case of the (4,0) brane studied in [18, 15], a non-vanishing $\mathcal{F}$ therefore necessarily requires mode depending gluing conditions.

To find the for the present case correct gluing matrix $N_n$ we consider the open string boundary condition (2.4) which gives the open-string operator identifications

$$a_n^I = -\left[\frac{\mathcal{F} + \frac{n}{\omega_n}}{\mathcal{F} - \frac{n}{\omega_n}}\right]^{IJ}\tilde{a}_n^J\,. \tag{3.12}$$

It is for example known from the literature on massive integrable boundary field theories how to translate this directly into the closed string boundary state picture [41].

This so called *crossing* which is usually done as an analytic continuation in the rapidity variable $\theta$ defined as $n = \sinh\theta_n$, is here simply given by

$$\theta_n \to \frac{i\pi}{2} - \theta_n\,, \qquad n \to i\omega_n\,, \qquad \omega_n \to -in \tag{3.13}$$

which leads to the bosonic gluing conditions

$$\left(a_n^I - \left[\frac{\mathcal{F} - \frac{\omega_n}{n}}{\mathcal{F} + \frac{\omega_n}{n}}\right]^{IJ} \tilde{a}_{-n}^J\right) ||B\rangle\rangle = 0\,. \tag{3.14}$$

Plugging (2.6) into this and comparing it with (3.6) gives

$$N_n = \frac{\mathcal{F} - \frac{\omega_n}{n}}{\mathcal{F} + \frac{\omega_n}{n}} = \frac{-(\omega_n - n) + (\omega_n + n)N}{(\omega_n + n) - (\omega_n - n)N} \tag{3.15}$$

with

$$N_{-n} = N_n\,; \qquad N_n^t N_{-n} = N_n^t N_n = 1\,. \tag{3.16}$$

The last relations make the bosonic gluing conditions self-consistent.

By translating (3.14) back into a relation between fields, one obtains

$$\left(\partial_\tau x^I - \mathcal{F}^{IJ}\partial_\sigma x^J\right) ||B\rangle\rangle = 0 \qquad (\tau = 0)\,. \tag{3.17}$$

The compared to (2.4) additional minus sign has its origin in the double Wick rotation $\sigma \to -i\sigma$ and $\tau \to i\tau$ which effectively takes place when changing from the open to the closed string channel.

Going with (3.15) into (3.10) one obtains

$$0 = (M\gamma^L M^t - \gamma^J N^{JL}) + \frac{\eta m}{2n}(\gamma^J \Pi M^t - M\gamma^J \Pi)(N^{JL} - \delta^{LJ}) \tag{3.18}$$

which implies the two following conditions for maximally supersymmetric branes

$$M\gamma^J M^t = \gamma^I N^{IJ} \tag{3.19}$$

$$\left(\delta^{KR} - N^{KR}\right)\left[\gamma^K \Pi M^t - M\gamma^k \Pi\right] = 0\,. \tag{3.20}$$

As we are currently only considering Neumann directions, the matrix $1 - N$ is invertible, such that we finally obtain the conditions

$$M\gamma^J M^t = \gamma^I N^{IJ} \tag{3.21}$$

$$\gamma^K = M\gamma^K \Pi M \Pi = \gamma^I N^{IK} M\Pi M\Pi \tag{3.22}$$

for possible gluing matrices of maximally supersymmetric static branes in the plane wave background.

So far we only considered contributions from nonzero modes to (3.1). As the "zero-modes" do not contain a $\sigma$-dependency in the closed string channel, $\mathcal{F}$ drops out for these modes and the previous considerations for a vanishing $\mathcal{F}$ for example in [15] remain unaltered: Commuting (3.1) with $x_0^I$ one obtains

$$\left(S_0^a + i\eta M^{ab}\tilde{S}_0^b\right) ||B\rangle\rangle = 0\,, \tag{3.23}$$

that is, the boundary state preserves 8 kinematical supersymmetries. Using this in (3.1), we are left with

$$\left(-i\eta P_0^I(\gamma^I M - M\gamma^I)\tilde{S}_0 - m x_0^I(\gamma^I \Pi - M\gamma^I \Pi M)\tilde{S}_0\right) ||B\rangle\rangle = 0\,. \tag{3.24}$$

For the Neumann directions this is solved with the standard requirement

$$P_0^I||B\rangle\rangle = 0\,. \tag{3.25}$$

For the Dirichlet directions, however, one has to have either

$$M\Pi M\Pi = 1\,, \tag{3.26}$$

which corresponds to a *class II* brane without gauge field excitations or

$$x_0^i||B\rangle\rangle = 0\,, \tag{3.27}$$

that is, the brane has to be restricted to the origin in transverse space.

## 4. Supersymmetric branes with nontrivial $\mathcal{F}$

The first condition (3.21) is identical to (2.10) for branes in flat space. As already mentioned above, it says that the 3 matrices $M_{\dot{a}\dot{b}}$, $M_{ab}$ and $N_{IJ}$ are related by SO(8)-triality, compare for example with [38]. This condition is explicitly solved by the formulas given in [38, 39] (with a slightly different normalization)

$$N_{IJ} = e^{\frac{1}{2}\Omega_{MN}\Sigma_{IJ}^{MN}} \tag{4.1}$$

and

$$M_{\dot{a}\dot{b}} = e^{\frac{1}{4}\Omega_{MN}\gamma_{\dot{a}\dot{b}}^{MN}}\,; \qquad M_{ab} = e^{\frac{1}{4}\Omega_{MN}\gamma_{ab}^{MN}} \tag{4.2}$$

with

$$\Sigma_{IJ}^{MN} = \delta_I^M\delta_J^N - \delta_I^N\delta_J^M\,; \qquad \gamma^{mn} = \frac{1}{2}\gamma^{[m}\gamma^{n]}\,. \tag{4.3}$$

The second condition (3.22) has no flat space analogue as it explicitly contains the matrix $\Pi$. For maximally supersymmetric branes this condition gives rise to some qualitative differences compared to flat space, where a nonzero boundary condensate does not give rise to any new constraints.

Before studying cases with nonzero magnetic fields in detail, it is easy to see that all the considerations so far are consistent with the previous works on branes in the plane wave background. Assuming a mode independent fermionic gluing condition as for example in [13, 14], one needs $M\Pi M\Pi = -1$ which furthermore gives with (3.22) $N = -1$, such that the bosonic gluing conditions finally reduce to the usual $N_n = N = -1$.

The maximally supersymmetric class II branes with $M\Pi M\Pi = 1$ are not contained in the previous discussion as the (0,0)-instanton does not have Neumann directions and as the (4,0), (0,4) branes couple necessarily to the flux $\mathcal{F}_{+I}$ ([10, 42]), a possibility to be discussed in the context of the $(4,2)$-brane with boundary condensate later on.

The previous discussion shows in addition that the compared to flat space new feature of mode dependent gluing conditions as in (3.3) is actually generic for (maximally supersymmetric) branes in the plane wave and not a speciality of the D-instanton.



JHEP06(2005)041

### 4.1 The (2,0), (0,2) branes

The cases of the (2,0) or (0,2)[2] (euclidean D1-) branes are solved as follows. Without loss of generality we choose the first two coordinates $I = 1, 2$ as Neumann directions. With this we obtain

$$N = \exp\left[\theta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right] = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{4.4}$$

and

$$M_{ab} = \exp\left[\frac{\theta}{2}\gamma^{12}_{ab}\right] = 1_{ab}\cos\frac{\theta}{2} + \gamma^{12}_{ab}\sin\frac{\theta}{2}\,. \tag{4.5}$$

Furthermore we have

$$[M, \Pi] = 0\,, \tag{4.6}$$

such that the second equation in (3.21) reads

$$\gamma^J = \gamma^I N^{IJ} M^2 = M\gamma^J M \leftrightarrow M^t\gamma^J = \gamma^J M \qquad (J = 1, 2)\,. \tag{4.7}$$

For the (2,0) case this is an identity without further conditions on $N$. Thus we can have arbitrary constant boundary condensates $\mathcal{F}$ on (euclidean) D1-branes without additional supersymmetry breaking.

As it should, the boundary condensate $\mathcal{F}$ somehow interpolates between the usual class I (2,0) brane and the (class II) (0,0) instanton. Choosing $\theta = 0$ the gluing matrices reduce to

$$K_n = \frac{\omega_n - \eta m\Pi}{n}\,; \qquad N_n = 1\,, \tag{4.8}$$

which are the conditions for the D-instanton and for $\theta = \pi$ we obtain

$$K_n = \gamma^{12}\,; \qquad N_n = -1\,, \tag{4.9}$$

which are the conditions for the usual (2,0) brane.

The boundary state of the (2,0) brane, its consistency with the open string channel description and further results from the interpolation to the instanton will be discussed in the next section.

### 4.2 The (3,1), (1,3) branes

Unlike the flat space case it is in the plane wave background impossible to turn on a boundary condensate on a (true) subspace of the brane world volume and still maintain maximal supersymmetry. This follows simply from the observation that if $N$ has an eigenvalue $-1$, the condition (3.22) immediately leads to the condition of class I branes and especially to $N \equiv -1$.

Even for a non-degenerate $\mathcal{F}$ the condition (3.22) is in general not solvable as the example of the (3,1) brane shows.[3] To study this case it is convenient to choose a coordinate

[2] For branes with nonzero magnetic fields we use the same labelling as for their $\mathcal{F} \to 0$ limits from [18, 15].

[3] The following analysis extends immediately to the case of (4,2), (2,4) branes without additional $\mathcal{F}^{-I}$ condensates.

JHEP06(2005)041

systems such that the antisymmetric $\Omega$ in (4.2) takes a particularly simple form. From the SO(4) $\times$ SO(4) background symmetry we have in this case only a SO(3) symmetry on the world volume which allows us to bring $\Omega$ to the form

$$\Omega = \begin{pmatrix} 0 & a & 0 & 0 \\ -a & 0 & 0 & b \\ 0 & 0 & 0 & c \\ 0 & -b & -c & 0 \end{pmatrix}. \tag{4.10}$$

(To block-diagonalise $\Omega$, that is, to set $b = 0$, a full SO(4)-rotation would generally be necessary.) By using (3.22) we can now show that a nontrivial boundary condensate on a flat $(3,1)$-brane is not consistent with maximal supersymmetry (as long as we do not change the gluing conditions (3.2) to allow non-static configurations, see below). Indeed, using $\Omega$ as above and aligning the brane along the 1,2,3,5 directions, we have

$$M = \exp\left[\frac{a}{2}\gamma^{12} + \frac{b}{2}\gamma^{25} + \frac{c}{2}\gamma^{35}\right]. \tag{4.11}$$

Evaluating (3.22) in the $I = 5$ direction gives $1 = \Pi M^2 \Pi = M^2$ which leads with (3.21) to $N^2 = 1$, that is, the case of a trivial boundary condensate $\mathcal{F} = 0$.

This observation, i.e. that it is impossible to turn on a gauge field condensate on a (3,1) brane and still maintain maximal supersymmetry, is related to the previous observation that the class I branes break all dynamical supersymmetries when removed from the origin of transverse space [13, 17]. Indeed, from the open string bosonic mode expansion (6.27) to be derived below, it can be seen that the zero modes along the Neumann directions with nontrivial $\mathcal{F}$ tend in the (well behaved) $\mathcal{F} \to \infty \Leftrightarrow \theta \to 0$ limit to the Dirichlet zero modes describing a brane removed from the origin.

From this it follows that only static (euclidean) branes related to the instanton (as discussed before) or the $(4,0), (0,4)$ branes (to be considered in the next section) can preserve 8 dynamical supersymmetries when boundary condensates are switched on and one does not allow for more general Dirichlet boundary conditions than in (3.2).

The probe brane approach of [10] actually suggests that the coupling of boundary condensates with the non-trivial background flux in the plane wave world volume action requires in a generic (especially non-static) case the introduction of deformed Dirichlet-conditions, corresponding to deformed D-brane embeddings. An example along these lines is the possibility to shift a lorentzian $(+,-,3,1)$ brane away from the origin when an appropriate $\mathcal{F}_{+I}$-flux is switched on, [10].

It would be interesting to understand whether a comparable deformation of the Dirichlet condition (3.2) would allow for a supersymmetric $(3,1)$ -brane with nontrivial boundary condensates, or more generally, whether other configurations on other branes than those discussed in this paper are possible in a maximally supersymmetric setting when deformed Dirichlet conditions are taken into account.

### 4.3 The $(4,2)$, $(2,4)$ branes with flux

As a second example of a brane with nontrivial gauge condensate $\mathcal{F}^{IJ} \neq 0$ we will consider in this section the $(4,2)$ case which is connected by the limiting process discussed above



JHEP06(2005)041

to the class II $(4,0)$ brane. As explained in [10], the $(4,0)$ brane couples to the nontrivial $F_5$ background flux in a way that the boundary condensate $\mathcal{F}_{+I} = \mathcal{F}^{-I}$ $(\mathcal{F}_{-I} = 0)$ is necessarily switched on to obey the equations of motion. The $\mathcal{F}^{-I}$-coupling alters the bosonic gluing conditions along the $I = 1,..4$ Neumann directions, but leaves the other gluing conditions as discussed before in section 3.

We will start with a (4,2) - brane and switch on a boundary condensate along the $A = 5, 6$ Neumann directions. Using the fermionic gluing matrix

$$\widehat{M} = \Pi \exp\left[\frac{\theta}{2}\gamma^{56}\right] \tag{4.12}$$

and employing the gluing conditions (3.2) and (3.4) as before, the condition (3.1)

$$\left(Q_{\dot{a}} + i\eta M_{\dot{a}\dot{b}}\tilde{Q}_{\dot{b}}\right) ||(4,2), \mathcal{F}^{-I}, \theta\rangle\rangle = 0 \tag{4.13}$$

uniquely determines the bosonic gluing conditions along the $I = 1, \ldots, 4$ directions to[4]

$$\left(\partial_+ X^I + \partial_- X^I - im\cos\frac{\theta}{2}X^I\right) ||(4,2), \mathcal{F}^{-I}, \theta\rangle\rangle = 0 \qquad (\tau = 0) \tag{4.14}$$

which in terms of modes reads

$$\left[\overline{a}^I_0 + \frac{1-\cos\frac{\theta}{2}}{1+\cos\frac{\theta}{2}} a^I_0\right] ||(4,2), \mathcal{F}^{-I}, \theta\rangle\rangle = 0 \tag{4.15}$$

and

$$\left[a^I_n + \left(\frac{\omega_n - m\cos\frac{\theta}{2}}{\omega_n + m\cos\frac{\theta}{2}}\right)\widetilde{a}^I_{-n}\right] ||(4,2), \mathcal{F}^{-I}, \theta\rangle\rangle = 0\,. \tag{4.16}$$

The gluing conditions (4.16) are in direct analogy to the (4,0) case discussed for example in [18, 15]. As for the case of the (2,0) brane, the boundary condensate $\mathcal{F}$ interpolates smoothly between the (4,2) and the (4,0) brane. It is worth noting that in the $\theta \to \pi$ limit not only $\mathcal{F}^{AB}$, but also $\mathcal{F}^{-I}$ tends to zero to exactly reproduce the class I setting of [13, 18].

## 5. Boundary states and cylinder diagrams

In this section the (2,0) brane boundary state will be determined. By using it (and the analogous state for a (4,2) brane with $\mathcal{F} \neq 0$), certain cylinder diagrams will be found by calculating the corresponding boundary state overlaps, generalizing the results of [14, 15].

### 5.1 The (2,0) boundary state

From the gluing conditions derived in the last section and the standard (anti-) commutation relations summarized for example in the appendix of [15], one can immediately write down

[4] We will give a more detailed derivation of this result in the open string setting in section 6.

the boundary state of the (2,0) brane with boundary condensate $\mathcal{F}$ at transverse position $\mathbf{y}=\mathbf{0}$ (compare for example with [13, 15, 11])

$$
\begin{aligned}
||(2,0),\mathbf{0},\eta,P^+\rangle\rangle &= \mathcal{N}_\theta^{(2,0)}\exp\left[\sum_{k=1}^{\infty}\left(\frac{1}{\omega_k}a^i_{-k}\tilde{a}^i_{-k}+\frac{N_k^{IJ}}{\omega_k}a^I_{-k}\tilde{a}^J_{-k}-i\eta K_k^{ab}S^a_{-k}\tilde{S}^b_{-k}\right)\right]\times\\
&\qquad\times\exp\left[-\frac{1}{2}\left[\frac{1+\eta M}{1-\eta M}\right]_{ab}\theta^a_L\theta^b_L-\right.\\
&\qquad\qquad\left.-\frac{1}{2}\left[\frac{1-\eta M}{1+\eta M}\right]_{ab}\overline{\theta}^a_R\overline{\theta}^b_R\right]e^{\frac{1}{2}a^i_0a^i_0-\frac{1}{2}a^I_0a^I_0}|0\rangle\,.
\end{aligned}
\tag{5.1}
$$

The state $|0\rangle$ is here given by the usual Fock space vacuum corresponding to the fixed light-cone momentum $P^+$. It is in particular annihilated by the fermionic zero modes $\overline{\theta}_L$ and $\theta_R$, as defined in the appendix A of [15]. The normalization factor $\mathcal{N}_\theta^{(2,0)}$ has still to be identified by a comparison with the open string one-loop calculation to be carried out in section 6.

#### 5.1.1 Cylinder diagrams

As described for example in [14], the cylinder diagram is given in terms of boundary states by the following overlap

$$
\mathcal{A}_{\overline{\eta},\eta,\theta}=\langle\langle(2,0),\mathbf{0},\overline{\eta},-P^+,\theta||e^{-2\pi tHP^+}||(2,0),\mathbf{0},\eta,P^+,\theta\rangle\rangle\,. \tag{5.2}
$$

Keeping in mind the different momenta $P^+$ for the in- and out-going boundary states and its consequences, the overlap can be evaluated by standard methods. One obtains for the brane/brane case $\eta=\overline{\eta}$

$$
\mathcal{A}_{\eta,\eta,\theta}=\left(\mathcal{N}_\theta^{(2,0)}\right)^*\mathcal{N}_\theta^{(2,0)} \tag{5.3}
$$

and for the brane/antibrane case $\overline{\eta}=-\eta$

$$
\mathcal{A}_{\eta,-\eta,\theta}=\frac{\left(\mathcal{N}_\theta^{(2,0)}\right)^*\mathcal{N}_\theta^{(2,0)}}{\left(2\sinh\left[m\pi\sin\frac{\theta}{2}\right]\right)^4}\frac{(g_2^{(m)}(q,\theta))^4}{(f_1^{(m)}(q))^8}, \tag{5.4}
$$

where $f_1^{(m)}(q)$ is defined as in [14] and $g_2^{(m)}(q,\theta)$ is the following deformation of the function $g_2^{(m)}(q)$ of [15]:

$$
\begin{aligned}
g_2^{(m)}(q,\theta) &= 2\sinh\left[m\pi\sin\frac{\theta}{2}\right]q^{-2\Delta_m}\sqrt{\left(1+\frac{\sin^2\frac{\theta}{2}}{(1-\cos\frac{\theta}{2})^2}\,q^m\right)\left(1+\frac{\sin^2\frac{\theta}{2}}{(1+\cos\frac{\theta}{2})^2}\,q^m\right)}\times\\
&\qquad\times\prod_{n=1}^{\infty}\left(1+q^{\omega_n}\frac{\omega_n-m\cos\frac{\theta}{2}}{\omega_n+m\cos\frac{\theta}{2}}\right)\left(1+q^{\omega_n}\frac{\omega_n+m\cos\frac{\theta}{2}}{\omega_n-m\cos\frac{\theta}{2}}\right)\\
&= 2\sinh\left[m\pi\sin\frac{\theta}{2}\right]q^{-2\Delta_m}\times
\end{aligned}
\tag{5.5}
$$

$$\times \prod_{n\in\mathbb{Z}} \sqrt{\left(1+q^{\omega_n}\frac{\omega_n - m\cos\frac{\theta}{2}}{\omega_n + m\cos\frac{\theta}{2}}\right)\left(1+q^{\omega_n}\frac{\omega_n + m\cos\frac{\theta}{2}}{\omega_n - m\cos\frac{\theta}{2}}\right)} . \tag{5.6}$$

Here the zero mode contributions of (5.5) might alternatively be written as

$$q^{-2\Delta_m}\frac{4\sinh\left[m\pi\sin\frac{\theta}{2}\right]}{\sin\frac{\theta}{2}}\sqrt{\left(\sin^2\frac{\theta}{4}+\cos^2\frac{\theta}{4}q^m\right)\left(\cos^2\frac{\theta}{4}+\sin^2\frac{\theta}{4}q^m\right)} , \tag{5.7}$$

which will be used below to study the $\theta \to 0$ limit.

Using the open-string result (6.56) to be derived later on, the boundary state normalization factor $\mathcal{N}_\theta^{(2,0)}$ is given up to a phase by

$$\mathcal{N}_\theta^{(2,0)} = 2\sinh\left[m\pi\sin\frac{\theta}{2}\right] , \tag{5.8}$$

which again reproduces the $(2,0)$ result of [14], but vanishes in the instanton limit. This is, however, not surprising, as the fermionic part of the boundary state (5.1) diverges in this limit to give altogether a smooth behavior of the different overlaps in both limiting cases.

The behavior under modular transformations of this $\theta$-dependent family of functions will be discussed in the appendix. It can be seen that this family connects the functions $f_2^{(m)}(q)$ and $g_2^{(m)}(q)$ defined in [14, 15]:

$$\lim_{\theta\to 0} g_2^{(m)}(q,\theta) = g_2^{(m)}(q) \tag{5.9}$$

$$\lim_{\theta\to \pi} g_2^{(m)}(q,\theta) = 2\sinh\left[m\pi\right]\left(f_2^{(m)}(q)\right)^2 \tag{5.10}$$

and that (5.3) and (5.4) reproduce the (closed string) results of [14, 15].

### 5.2 (4,2)-(0,2) - overlap

As an example of an overlap containing the (4,2) boundary state with nonzero fluxes $\mathcal{F}^{AB}$ $(A,B=5,6)$ and $\mathcal{F}^{-I}$ $(I=1\dots 4)$ we consider here the cylinder diagram with a (0,2) - (anti-) brane with the same gauge field strength $\mathcal{F}^{AB}$ on its world-volume:

$$\mathcal{B}_{\eta,\theta} = \langle\langle (0,2), \mathbf{0}, \eta, -P^+, \theta || e^{-2\pi t H P^+} || (4,2), \mathbf{0}, P^+, \theta, \mathcal{F}^{-I}\rangle\rangle . \tag{5.11}$$

For the overlap with the (0,2) - brane ($\eta = 1$) we find

$$\mathcal{B}_{(\eta=1),\theta} = \mathcal{N}_\theta^{(0,2)}\mathcal{N}_\theta^{(4,2)}\left(\frac{\prod_{n=1}^\infty (1-q^{\omega_n})}{\prod_{n=1}^\infty\left(1+\frac{\omega_n - m\cos\frac{\theta}{2}}{\omega_n + m\cos\frac{\theta}{2}}q^{\omega_n}\right)}\right)^4 \tag{5.12}$$

and for the antibrane ($\eta = -1$)

$$\mathcal{B}_{(\eta=-1),\theta} = \mathcal{N}_\theta^{(0,2)}\mathcal{N}_\theta^{(4,2)}\left(\frac{\prod_{n=1}^\infty\left(1+\frac{\omega_n + m\cos\frac{\theta}{2}}{\omega_n - m\cos\frac{\theta}{2}}q^{\omega_n}\right)}{\prod_{n=1}^\infty (1-q^{\omega_n})}\right)^4 , \tag{5.13}$$



JHEP06(2005)041

where the zero-mode contributions as for example from the bosons ($I = 1, \ldots, 4$)

$$\left(1 + \frac{1 - \cos\frac{\theta}{2}}{1 + \cos\frac{\theta}{2}} q^m\right)^{-2} \tag{5.14}$$

cancel out with the corresponding fermionic contributions.

Besides the product representation of the $f_1^{(m)}$−function from [14], the appearing special functions are essentially given by different halves of the function $g_2^{(m)}$ defined in (5.6), which itself have a good behavior under modular transformations. A comparable result was found in [15] for the (4,0)-(2,0) overlap without gauge-field excitations.

## 6. Open string description

In this section we will study the previously mentioned branes from the open string point of view. First, we will consider open string (dynamical) supersymmetries in general and reproduce the conditions (3.21) and (3.22) derived beforehand in the boundary state approach. After that, we will give a detailed treatment of open strings in between $(2, 0)$ branes with $\mathcal{F} \neq 0$ and compare the results with those from the closed string picture.

The bosonic and fermionic open string equations of motion are the same as in the closed string sector, that is [2, 3]

$$\left(\partial_+\partial_- + \widehat{m}^2\right) X^s = \left(\partial_\tau^2 - \partial_\sigma^2 + \widehat{m}^2\right) X^s = 0 \tag{6.1}$$

for the bosons and

$$\partial_+ S = \widehat{m}\Pi\widetilde{S}\,; \qquad \partial_-\widetilde{S} = -\widehat{m}\Pi S \tag{6.2}$$

for the fermions. As explained in detail in [14, 15], the open string mass parameter $\widehat{m}$ appropriate for the light-cone gauge description of instantonic branes is here given by

$$\widehat{m} = \mu X^+ \tag{6.3}$$

instead of

$$m = 2\pi\mu P^+ \tag{6.4}$$

which is used in the closed string sector. The bosonic boundary conditions for the case of a non-vanishing boundary condensate $\mathcal{F}$ are as described in section 2

$$\partial_\sigma X^I + \mathcal{F}^{IJ}\partial_\tau X^J = 0\,; \qquad \sigma = 0, \pi \tag{6.5}$$

for the Neumann and

$$X^i = y^i_\sigma \qquad \sigma = 0\,, \pi \tag{6.6}$$

for the Dirichlet directions. For the fermions we use furthermore

$$S(\tau, \sigma = 0) = \widehat{M}\widetilde{S}(\tau, \sigma = 0)\,, \qquad S(\tau, \sigma = \pi) = \eta\widehat{M}\widetilde{S}(\tau, \sigma = \pi) \tag{6.7}$$

where $\eta = \pm 1$ distinguishes between the case of a brane-brane or a brane-antibrane pair.

### 6.1 Open string supersymmetries

The dynamical supersymmetries in the closed string sector follow from the conserved currents [2, 18]

$$Q^\tau = \partial_- X^s \gamma^s S - m X^s \gamma^s \Pi \widetilde{S} \tag{6.8}$$

$$Q^\sigma = \partial_- X^s \gamma^s S + m X^s \gamma^s \Pi \widetilde{S} \tag{6.9}$$

$$\widetilde{Q}^\tau = \partial_+ X^s \gamma^s \widetilde{S} + m X^s \gamma^s \Pi S \tag{6.10}$$

$$\widetilde{Q}^\sigma = -\partial_+ X^s \gamma^s \widetilde{S} + m X^s \gamma^s \Pi S \tag{6.11}$$

from which the (conserved) supercharges are obtained in the usual way to, for example,

$$\sqrt{2P^+} Q_{\dot\alpha} = \frac{1}{2\pi} \int_0^{2\pi} d\sigma \left( \partial_- X^s \gamma^s S - m X^s \gamma^s \Pi \widetilde{S} \right)_{\dot\alpha} . \tag{6.12}$$

The conserved supersymmetries in the open string sector are deduced from this as a suitable linear combination

$$Q_{\text{open}} = Q - K\widetilde{Q} \tag{6.13}$$

of the for open strings generally time-*dependent* charges $Q$ and $\widetilde{Q}$ with a so far undetermined constant SO(8)-spinor matrix $K$.

From the definition of (6.13) in terms of (6.8) and (6.10) it follows immediately that the open string supercharges (6.13) become time independent iff the following boundary terms vanish

$$\left( \left[ \partial_- X^s \gamma^s S + m X^s \gamma^s \Pi \widetilde{S} \right] - K \left[ -\partial_+ X^s \gamma^s \widetilde{S} + m X^s \gamma^s \Pi S \right] \right) \Big|_{\sigma=0}^{\pi} . \tag{6.14}$$

Separating this into contributions from Neumann and Dirichlet directions and furthermore using the gluing conditions (2.5) and (6.7), we obtain the conditions

$$0 = \partial_- X^I \left( \gamma^I \widehat{M} - N^{JI} K \gamma^J \right) \widetilde{S} + m X^I \left( \gamma^I \Pi - K \gamma^I \Pi \widehat{M} \right) \widetilde{S} \Big|_{\sigma=0}^{\pi} \tag{6.15}$$

from the Neumann and

$$0 = \partial_- X^i \left( \gamma^i \widehat{M} - K \gamma^i \right) \widetilde{S} + m X^i \left( \gamma^i \Pi - K \gamma^i \Pi \widehat{M} \right) \widetilde{S} \Big|_{\sigma=0}^{\pi} \tag{6.16}$$

from the Dirichlet directions. These conditions require then in particular

$$\gamma^I N^{IJ} = K^t \gamma^J \widehat{M} ; \qquad \gamma^I = K^t \gamma^I \Pi \widehat{M}^T \Pi ; \qquad \gamma^i \widehat{M} - K \gamma^i = 0 . \tag{6.17}$$

Comparing this with (3.21) and (3.22), it leads to $K = \widehat{M} = M^t$. As long as $\Pi \widehat{M} \Pi \widehat{M} \neq 1$ (for non-class II branes), we furthermore have to choose $y^i_\sigma = 0$, that is, to place the branes at the origin of transverse space to fulfill the conditions imposed by the Dirichlet directions.

The previous considerations lead for a (2,0) brane with $\mathcal{F} \neq 0$ to the use of the following matrix

$$\widehat{M} = M^t = \exp\left[ -\frac{\theta}{2} \gamma^{12} \right] \tag{6.18}$$



JHEP06(2005)041

in (6.7). In the conventions of [17, 10] this actually corresponds to an interpolation to the anti (2,0) brane, as $\widehat{M} \to -\gamma^{12}$ for $\theta \to \pi$. But this is simply the usual sign ambiguity in between the open — and closed string picture quantities and we will still refer to this as the (2,0) brane.

The conserved supercharges corresponding to the choice of $\widehat{M}$ as given above again interpolate between the instanton and the (2,0) supercharges appearing respectively in [15] and [17, 18] where the combinations $Q_{(0,0)} = Q - \widetilde{Q}$ were used for the instanton and $Q_{(2,0)} = Q + \gamma^{12}\widetilde{Q}$ for the $(2,0)$ - brane.

#### 6.1.1 The $(4,2)$ brane with $\mathcal{F}^{-I} \neq 0$

For the open string description of the $(4,2)$ brane with boundary condensates we use the following Ansatz for the boundary conditions at $\sigma = 0, \pi$:

$$\partial_+ X^i + \partial_- X^i = 0\,; \qquad X^i = 0\,; \qquad i = 7, 8 \tag{6.19}$$

$$\partial_+ X^A + N^{AB}\partial_- X^B = 0\,; \qquad A, B = 5, 6 \tag{6.20}$$

$$\partial_+ X^I - \partial_- X^I + \alpha m X^I = 0\,; \qquad I = 1, \dots 4 \tag{6.21}$$

$$S = \widehat{M}\widetilde{S}\,. \tag{6.22}$$

Going with this into equation (6.14), one obtains for the Dirichlet ($i = 7, 8$) and the Neumann ($A = 5, 6$) directions exactly the same conditions as in (3.21) and (3.22) when using $K = \widehat{M}$ in (6.13) as before. These conditions are in particular solved by the matrix

$$\widehat{M} = \Pi \exp\left[-\frac{\theta}{2}\gamma^{56}\right] \tag{6.23}$$

which gives in the $\theta \to \pi$ limit the correct gluing matrix for the $(4,2)$ brane (for which then actually *all* $\mathcal{F}$ components will be seen to vanish).

For $I = 1, \dots 4$ the requirement for eight conserved dynamical supersymmetries becomes

$$0 = \partial_- X^I \left(\gamma^I M + M\gamma^I\right)\widetilde{S} + m X^I M \left(M^t \gamma^I \Pi - \gamma^I \Pi M - \alpha \gamma^I\right)\Big|_{\sigma=0}^{\pi} \tag{6.24}$$

which vanishes with (6.23) when using $\alpha = 2m\cos\frac{\theta}{2}$. The last equality determines the strength of the $\mathcal{F}^{-I}$ components in dependency of the 'transverse' field-strength $\mathcal{F}^{AB}$.

### 6.2 The (2,0) - open string description

In the following we will give a detailed open string treatment of the (2,0) brane with nontrivial boundary condensate. First we will derive the relevant bosonic and fermionic mode expansions and determine the light-cone gauge hamiltonian. After a brief discussion of the canonical quantization, we will finally calculate some open string partition functions and relate them to closed string boundary state overlaps calculated beforehand by modular transformations. It is worth mentioning that the analogous case of a (4,2) brane with flux discussed already in the closed string sector, is dealt with by essentially the same calculations, such that we will omit the details here.



JHEP06(2005)041

### 6.2.1 Bosons

The most general solution of (6.1) is given by

$$x^s(\sigma,\tau) = A^s \sin(\widehat{m}\tau) + \tilde{A}^s \cos(\widehat{m}\tau) + B^s \cosh(\widehat{m}\sigma) + \tilde{B}^s \sinh(\widehat{m}\sigma) + \\ +i \sum_{n,\omega_n \in \mathbb{C}\setminus\{0\}} \frac{1}{\omega_n}\left(A^s_n e^{-i(\omega_n\tau - n\sigma)} + \tilde{A}^s_n e^{-i(\omega_n\tau + n\sigma)}\right) \tag{6.25}$$

with $\omega_n^2 = n^2 + \widehat{m}^2$. Using this, the mode expansion for the Dirichlet directions of strings ending on branes at the origin of transverse space equals

$$X^i(\tau,\sigma) = -2 \sum_{n\in\mathbb{Z}\setminus\{0\}} \frac{e^{-i\omega_n\tau}}{\omega_n} \alpha^i_n \sin(n\sigma)\,; \qquad \omega_n = \mathrm{sgn}(n)\sqrt{n^2+\widehat{m}^2}\,. \tag{6.26}$$

For the Neumann directions, on the other hand, one has

$$X^I(\sigma,\tau) = e^{-i\widehat{m}\sin\frac{\theta}{2}\tau} \exp\left[iJ\widehat{m}\cos\frac{\theta}{2}\sigma\right] a^I + e^{i\widehat{m}\sin\frac{\theta}{2}\tau} \exp\left[-iJ\widehat{m}\cos\frac{\theta}{2}\sigma\right] a^{\dagger I} + \\ +i \sum_{n\in\mathbb{Z}\setminus\{0\}} \frac{e^{-i\omega_n\tau}}{\omega_n}\left[\alpha^I_n e^{in\sigma} + \tilde{\alpha}^I_n e^{-in\sigma}\right] \tag{6.27}$$

with

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \qquad \tilde{\alpha}^I_n = -\frac{\mathcal{F} - \frac{n}{\omega_n}}{\mathcal{F} + \frac{n}{\omega_n}}\alpha_n\,; \qquad \mathcal{F} = \begin{pmatrix} 0 & f \\ -f & 0 \end{pmatrix}; \qquad f = \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}\,, \tag{6.28}$$

where the last equation follows from comparing (2.6) with (4.4) which gives

$$N = \frac{1}{1+f^2}\begin{pmatrix} -(1-f^2) & 2f \\ -2f & -(1-f^2) \end{pmatrix}. \tag{6.29}$$

The first two terms in (6.27) which are the substitute for the otherwise absent zero modes in the case of a boundary condensate, correspond to modes $n$ for which the matrix $\mathcal{F} \pm \frac{n}{\omega_n}$ appearing in (6.28) is degenerate, that is, when

$$0 = \det\left(\mathcal{F} \pm \frac{n}{\omega_n}\right) = f^2 + \left(\frac{n}{\omega_n}\right)^2 \Leftrightarrow n = \pm i\widehat{m}\cos\frac{\theta}{2}\,. \tag{6.30}$$

As already mentioned in [21] where this (bosonic) mode expansion for $\mathcal{F} \neq 0$ was written down in different conventions which more closely resemble the usual flat space description as for example in [43], but which have a singular behavior in the $\theta \to 0$ limit, these two extra terms fulfill the condition (2.4) actually for all $\sigma$, not only on the boundary. In the limits $\theta \to \pi, 0$ these terms tend to (redefinitions of) the usual "zero"-modes of the $(2,0)$-brane or the instanton.

### 6.2.2 Fermions

The most general solutions of the fermionic equations of motion (6.2) are given by [15]

$$\begin{aligned} S(\sigma,\tau) &= S\cos(\widehat{m}\tau)+\Pi\tilde{S}\sin(\widehat{m}\tau)+T\cosh(\widehat{m}\sigma)+\Pi\tilde{T}\sinh(\widehat{m}\sigma)+\\ &+\sum_{n,\omega_m\in\mathbb{C}\backslash\{0\}} c_n\left[S_n e^{-i(\omega_n\tau-n\sigma)}+\frac{i}{\widehat{m}}(\omega_n-n)\Pi\tilde{S}_n e^{-i(\omega_n\tau+n\sigma)}\right] \end{aligned} \tag{6.31}$$

$$\begin{aligned} \tilde{S}(\tau,\sigma) &= -\Pi S\sin(\widehat{m}\tau)+\tilde{S}\cos(\widehat{m}\tau)+T\cosh(\widehat{m}\sigma)+\Pi T\sinh(\widehat{m}\sigma)+\\ &+\sum_{n,\omega_m\in\mathbb{C}\backslash\{0\}} c_n\left[\tilde{S}_n e^{-i(\omega_n\tau+n\sigma)}-\frac{i}{\widehat{m}}(\omega_n-n)\Pi S_n e^{-i(\omega_n\tau-n\sigma)}\right] \end{aligned} \tag{6.32}$$

with the boundary conditions (6.7) with $\widehat{M}=\exp\left[-\frac{\theta}{2}\gamma^{12}\right]$.

For a nontrivial $\widehat{M}$ with $\theta\in(0,\pi)$ the boundary conditions (6.7) lead in both cases $\eta=\pm1$ to vanishing 'zero'-modes $S=T=\tilde{S}=\tilde{T}=0$. The conditions for the nonzero modes read

$$\left(\mathbb{1}+\frac{i}{\widehat{m}}(\omega_n-n)\widehat{M}\Pi\right)S_n = \left(\widehat{M}-\frac{i}{\widehat{m}}(\omega_n-n)\Pi\right)\tilde{S}_n \tag{6.33}$$

$$\left(\mathbb{1}+\eta\frac{i}{\widehat{m}}(\omega_n-n)\widehat{M}\Pi\right)S_n = \left(\eta\widehat{M}-\frac{i}{\widehat{m}}(\omega_n-n)\Pi\right)e^{-2\pi i n}\tilde{S}_n\,. \tag{6.34}$$

**Brane - brane, $\eta=1$.** Using (6.33) and (6.34), the mode expansions for strings stretching between a brane-brane pair can be determined to be

$$\begin{aligned} S(\tau,\sigma) &= \frac{\mathbb{1}+\Pi}{2}\exp\left[+\widehat{m}\sin\frac{\theta}{2}\tau\gamma^{12}\right]S_0 e^{\widehat{m}\cos\frac{\theta}{2}\sigma}+\\ &+\frac{\mathbb{1}-\Pi}{2}\exp\left[-\widehat{m}\sin\frac{\theta}{2}\tau\gamma^{12}\right]\widehat{M}S_0 e^{-\widehat{m}\cos\frac{\theta}{2}\sigma}+\\ &+\sum_{n\in\mathbb{Z}\backslash\{0\}} c_n\left[S_n e^{-i(\omega_n\tau-n\sigma)}+\frac{i}{\widehat{m}}(\omega_n-n)\Pi\tilde{S}_n e^{-i(\omega_n\tau+n\sigma)}\right]\,; \end{aligned} \tag{6.35}$$

$$\begin{aligned} \tilde{S}(\tau,\sigma) &= \frac{\mathbb{1}+\Pi}{2}\exp\left[+\widehat{m}\sin\frac{\theta}{2}\tau\gamma^{12}\right]\widehat{M}^t S_0 e^{\widehat{m}\cos\frac{\theta}{2}\sigma}+\\ &+\frac{\mathbb{1}-\Pi}{2}\exp\left[-\widehat{m}\sin\frac{\theta}{2}\tau\gamma^{12}\right]S_0 e^{-\widehat{m}\cos\frac{\theta}{2}\sigma}+\\ &+\sum_{n\in\mathbb{Z}\backslash\{0\}} c_n\left[\tilde{S}_n e^{-i(\omega_n\tau+n\sigma)}-\frac{i}{\widehat{m}}(\omega_n-n)\Pi S_n e^{-i(\omega_n\tau-n\sigma)}\right] \end{aligned} \tag{6.36}$$

with $\omega_n=\mathrm{sgn}(n)\sqrt{n^2+\widehat{m}^2}$, $c_n=\frac{\widehat{m}}{\sqrt{2\omega_n(\omega_n-n)}}$, $-2\sin\frac{\theta}{2}\gamma^{12}=\widehat{M}-\widehat{M}^t$ and the operator identifications

$$\tilde{S}_n=\frac{\left(\mathbb{1}+\frac{i}{\widehat{m}}(\omega_n-n)\widehat{M}\Pi\right)}{\left(\widehat{M}-\frac{i}{\widehat{m}}(\omega_n-n)\Pi\right)}S_n\,. \tag{6.37}$$

The first two terms in each expansion correspond as in the bosonic case to $n=\pm i\widehat{m}\cos\frac{\theta}{2}$ for which the matrices in (6.33) and (6.34) are degenerate. As before, these "zero"-modes actually fulfill the fermionic boundary conditions for all $\sigma\in[0,\pi]$ and not only on the boundary.

**Brane-antibrane, $\eta = -1$.** Contrary to the situation before, there are no extra non-zero mode contribution in the case of a open string joining a brane and an antibrane (with the same gauge condensate $\mathcal{F}$). This follows directly by combining (6.33) and (6.34) to

$$\left(n - i\widehat{m}\cos\frac{\theta}{2}\Pi\right)\tilde{S}_n = -\left(n + i\widehat{m}\cos\frac{\theta}{2}\Pi\right)e^{-2\pi in}\tilde{S}_n \tag{6.38}$$

which gives $\tilde{S}_{n=\pm i\widehat{m}\cos\frac{\theta}{2}} = 0$.

The identification between the nonzero-modes $\tilde{S}_n$ and $S_n$ is still given by (6.37) which again follows from (6.33). To simultaneously fulfill the second condition (6.34), the moding $n$ has to fulfill the following equation

$$n \in P_\theta^+ : \qquad \frac{n + i\widehat{m}\cos\frac{\theta}{2}}{n - i\widehat{m}\cos\frac{\theta}{2}} = -e^{2\pi in}; \qquad n \neq 0 \tag{6.39}$$

for the $S_n^+ = \frac{\mathbb{1}+\Pi}{2}S_n$ modes and

$$n \in P_\theta^- : \qquad \frac{n - i\widehat{m}\cos\frac{\theta}{2}}{n + i\widehat{m}\cos\frac{\theta}{2}} = -e^{2\pi in}; \qquad n \neq 0 \tag{6.40}$$

for the $S_n^- = \frac{\mathbb{1}-\Pi}{2}S_n$ modes. This is in direct analogy to the $(0,0) - \overline{(0,0)}$ situation described in [15]. Both equations have infinitely many solutions on the real axis and for small $\widehat{m}$ all of them are being close to the flat space case of half integers. From $\widehat{m}\cos\frac{\theta}{2} > \frac{1}{\pi}$ on, however, two solutions of $P_\theta^-$ become imaginary. This is somewhat in analogy to the additional 'zero-modes' for a string stretching between two branes of the same kind with nonzero flux as discussed before.

### 6.2.3 The light-cone gauge hamiltonian

The light-cone gauge hamiltonian is given by [2, 3]

$$\frac{X^+}{2\pi}H^{\text{open}} = \frac{1}{4}\int_0^\pi d\sigma\left(\dot{X}^2 + X'^2 + \widehat{m}^2X^2\right) + \frac{i}{2}\int_0^\pi d\sigma\left(S\dot{S} + \tilde{S}\dot{\tilde{S}}\right) \tag{6.41}$$

and has the following expression in terms of modes

$$\begin{aligned}\frac{X^+}{2\pi}H^{\text{open}} = {} & \frac{\widehat{m}}{2\cos\frac{\theta}{2}}\sinh\left[\widehat{m}\pi\cos\frac{\theta}{2}\right]\left(ae^{-i\pi\widehat{m}J\cos\frac{\theta}{2}}a^\dagger + a^\dagger e^{i\pi\widehat{m}J\cos\frac{\theta}{2}}a\right) + \\ & + i\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\sinh\left[\widehat{m}\pi\cos\frac{\theta}{2}\right]\left(S_0\frac{1-\Pi}{2}\gamma^{12}S_0e^{-\widehat{m}\cos\frac{\theta}{2}\pi} - S_0\frac{1+\Pi}{2}\gamma^{12}S_0e^{\widehat{m}\cos\frac{\theta}{2}\pi}\right) + \\ & + \pi\sum_{n\neq 0}\left(\alpha_{-n}^I\alpha_n^I + \alpha_{-n}^i\alpha_n^i + \omega_nS_{-n}S_n\right)\end{aligned} \tag{6.42}$$

for a open string stretching between a brane-brane pair at the same transverse position $\mathbf{y} = \mathbf{0}$. In the brane-antibrane case the fermionic zero modes $S_0$ are absent and the fermionic nonzero modes have to fulfill either (6.39) or (6.40). In this case there is furthermore a nontrivial normal ordering constant to be discussed briefly below which is (apart from possible zero mode contributions) absent in the first case due to supersymmetry.

#### 6.2.4 Quantization

The quantization proceeds in the usual way. By stressing that in our case $\mathcal{F} = F, B = 0$, it is clear that in particular the fermionic canonical conjugated momenta are unaffected by the boundary condensates. Requiring therefore the equal time (anti-) commutation relations (as discussed with further details for the bosons in [21, 43])

$$[X^I(\tau,\sigma), P^J(\tau,\sigma')] = i\delta^{IJ}\delta(\sigma-\sigma')\,, \tag{6.43}$$

$$\{S^a(\tau,\sigma), S^b(\tau,\sigma')\} = \{\tilde{S}^a(\tau,\sigma), \tilde{S}^b(\tau,\sigma')\} = 2\pi\delta(\sigma-\sigma')\delta^{ab}\,, \tag{6.44}$$

$$\{S^a(\tau,\sigma), \tilde{S}^b(\tau,\sigma')\} = 0\,, \tag{6.45}$$

one obtains the following relations for the modes.

For the bosons

$$[\alpha^i_n, \alpha^j_m] = \omega_n \delta_{m+n}\delta^{ij} \tag{6.46}$$

$$[\alpha^I_n, \alpha^J_m] = \omega_n \delta_{m+n}\delta^{IJ} \tag{6.47}$$

$$[a^I, a^{\dagger J}] = \pi\sin\theta\left[\frac{\cosh(\widehat{m}\pi\cos\frac{\theta}{2})}{\sinh(\widehat{m}\pi\cos\frac{\theta}{2})} - iJ\right]^{IJ} \tag{6.48}$$

$$= \frac{\pi\sin\theta}{\sinh\left[\widehat{m}\pi\cos\frac{\theta}{2}\right]}\exp\left[-iJ\widehat{m}\pi\cos\frac{\theta}{2}\right]^{IJ} \tag{6.49}$$

and for the fermions

$$\{S^a_0, S^b_0\} = \frac{\widehat{m}\pi\cos\frac{\theta}{2}}{\sinh\left[\widehat{m}\pi\cos\frac{\theta}{2}\right]}\left(\frac{1+\Pi}{2}e^{-\pi\widehat{m}\cos\frac{\theta}{2}} + \frac{1-\Pi}{2}e^{\pi\widehat{m}\cos\frac{\theta}{2}}\right)^{ab} \tag{6.50}$$

$$\{S^a_n, S^b_m\} = \delta_{n+m}\delta^{ab}\,. \tag{6.51}$$

Some details of the derivations will be given in the appendix.

#### 6.2.5 Partition functions

In this section the open string partition functions for strings stretching between (2,0) branes with flux will be calculated for the cases of a brane-brane and a brane-antibrane pair.

As discussed in [27, 14], these partition functions are given by

$$Z(\widetilde{t}) = \mathrm{Tr}\exp\left[-\frac{X^+}{2\pi}H^{\mathrm{open}}\widetilde{t}\right]\,, \tag{6.52}$$

where the trace runs over the open string Hilbert spaces as (implicitly) determined in the previous subsection.

For the brane-brane pair, the normal ordered contributions of the bosonic zero modes in (6.42) are for example given by

$$\frac{\widehat{m}\sinh\left[\widehat{m}\pi\cos\frac{\theta}{2}\right]}{\cos\frac{\theta}{2}}a^\dagger\exp\left[i\pi\widehat{m}J\cos\frac{\theta}{2}\right]a + 2\pi\widehat{m}\sin\frac{\theta}{2} \tag{6.53}$$

and are therefore leading to a factor

$$\frac{1}{\left(2\sinh\left[\widehat{m}\pi\widetilde{t}\sin\frac{\theta}{2}\right]\right)^2} \tag{6.54}$$

as contribution to (6.52).

Defining the fermionic vacuum by the requirement that it is for example annihilated by the combinations $A^l = S_0^{1+2l} + iS_0^{2+2l}$, $l = 0,\dots,3$, the fermionic zero modes furthermore give rise to the factor

$$\left(2\sinh\left[\widehat{m}\pi\widetilde{t}\sin\frac{\theta}{2}\right]\right)^4 . \tag{6.55}$$

Together with the nonzero-mode contributions which are evaluated as for example in [15], one obtains the following open string partition functions

$$Z_{\eta,\eta,\theta}(\widetilde{t}) = \left(2\sinh\left[\widehat{m}\pi\widetilde{t}\sin\frac{\theta}{2}\right]\right)^2 \tag{6.56}$$

and

$$Z_{\eta,-\eta,\theta}(\widetilde{t}) = \frac{1}{\left(2\sinh\left[\widehat{m}\pi\widetilde{t}\sin\frac{\theta}{2}\right]\right)^2}\frac{\left(\widehat{g}_4{}^{(\widehat{m})}(\widetilde{t},\theta)\right)^4}{\left(f_1^{(\widehat{m})}(\widetilde{t})\right)^8} \tag{6.57}$$

for open strings stretching between a brane-brane or a brane-antibrane pair. The function $f_1^{(\widehat{m})}(\widetilde{t})$ is here defined as in [14, 15] and we have furthermore set

$$\widehat{g}_4^{(\widehat{m})}(\widetilde{t},\theta) = 2\sinh\left[\widehat{m}\pi\widetilde{t}\right]\widetilde{q}^{-\overline{\Delta}_{\widehat{m},\theta}+\frac{\widehat{m}}{2}(1-\sin\frac{\theta}{2})}\prod_{\lambda\in P_\theta^+}\sqrt{\left(1-\widetilde{q}^{\sqrt{\lambda^2+\widehat{m}^2}}\right)}\prod_{\lambda\in P_\theta^-}\sqrt{\left(1-\widetilde{q}^{\sqrt{\lambda^2+\widehat{m}^2}}\right)} \tag{6.58}$$

as $\theta$-dependent generalization of the function $g_2^{(\widehat{m})}$ appearing in [15]. The offset $\overline{\Delta}_{\widehat{m},\theta}$ is essentially determined by the normal ordering constant in the light-cone gauge hamiltonian. Its explicit form will be given in the appendix.

As in the closed string picture, this family of functions reproduces the results of [15] in the limits $\theta \to 0, \pi$:

$$\lim_{\theta\to 0}\widehat{g}_4^{(\widehat{m})}(\widetilde{t},\theta) = \widehat{g}_4^{(\widehat{m})}(\widetilde{t}) \tag{6.59}$$

$$\lim_{\theta\to \pi}\widehat{g}_4^{(\widehat{m})}(\widetilde{t},\theta) = 2\sinh\left[m\pi\right]\left(f_4^{(\widehat{m})}(\widetilde{q})\right)^2 , \tag{6.60}$$

which is in particular consistent with the modular transformation properties discussed in [14, 15].

## 7. Conclusion

In this paper we have derived the general conditions for static maximally supersymmetric branes with nontrivial $\mathcal{F}^{IJ}$ world-volume fluxes in the plane wave background. Both, the open - and closed string picture gave rise to the same results, generalizing the findings of [13, 17] and [10].

In a next step we have solved these conditions and found that the constant magnetic boundary fields give rise to new (continuous) families of maximally supersymmetric branes which are connected in the limit of infinite field strengths to the previously classified class II branes.

In contradistinction to flat space, magnetic fields cannot be turned on on every maximally supersymmetric brane without breaking some further supersymmetries. We argued that this behaviour is directly related to the previous observation from [13, 17] that class I branes are only supersymmetric when placed at the origin of transverse space.

After constructing boundary states and determining certain overlaps, we have shown in addition that the new branes pass the important open/closed duality consistency check by determining open string partition functions and demonstrating their equivalence with the closed string results. This final step involved a new family of modular functions, generalizing and connecting the results of [14, 15].


## Acknowledgments

It is a pleasure to thank Matthias Gaberdiel for encouragement and support and Carlo Albert, Stefan Fredenhagen, Peter Kaste and Martin Weidner for discussions. This research is partially supported by the Swiss National Science Foundation and the European Network 'ForcesUniverse' (MRTN-CT-2004-005104).


## A. Quantization

In this section some details of the canonical quantization for the fermionic and bosonic (open string) degrees of freedom will be given.

### A.1 Fermions

Using the relation $\{S^a_n, S^b_m\} = \delta_{n+m}\delta^{ab}$ we obtain with (6.35)

$$\begin{aligned}\{S(\tau,\sigma), S(\tau,\sigma')\} &= e^{\widehat{m}\cos\frac{\theta}{2}(\sigma+\sigma')}\frac{1+\Pi}{2}\{S_0,S_0\} + e^{-\widehat{m}\cos\frac{\theta}{2}(\sigma+\sigma')}\frac{1-\Pi}{2}\{S_0,S_0\} + \\ &\quad + \sum_{n\neq 0}\left(e^{in(\sigma-\sigma')} - \frac{\widehat{m}i}{2\omega_n}\Pi(K^t_{-n}+K_{-n})e^{in(\sigma+\sigma')}\right), \qquad \text{(A.1)}\end{aligned}$$

where $K_n$ is the matrix appearing in (6.37). Using

$$K^t_{-n} + K_{-n} = \frac{2\omega_n\cos\frac{\theta}{2}}{n + i\widehat{m}\Pi\cos\frac{\theta}{2}} = 2\omega_n\cos\frac{\theta}{2} \qquad \frac{n - i\widehat{m}\Pi\cos\frac{\theta}{2}}{n^2+\widehat{m}^2\cos^2\frac{\theta}{2}} \qquad \text{(A.2)}$$

the sum in (A.1) becomes

$$\sum_{n\in\mathbb{Z}}\left(e^{in(\sigma-\sigma')} - i\widehat{m}\cos\frac{\theta}{2}\Pi\frac{ne^{in(\sigma+\sigma')}}{n^2+\widehat{m}^2\cos^2\frac{\theta}{2}} - \widehat{m}^2\cos^2\frac{\theta}{2}\frac{e^{in(\sigma+\sigma')}}{n^2+\widehat{m}^2\cos^2\frac{\theta}{2}}\right). \qquad \text{(A.3)}$$

The infinite sums can be evaluated as contour integrals as in [15], giving for example

$$\sum_{n\in\mathbb{Z}} \frac{n e^{in(\sigma+\sigma')}}{n^2+\widehat{m}^2\cos^2\frac{\theta}{2}} = -\int_C \frac{1}{1-e^{2\pi i z}} \frac{z}{z^2+\widehat{m}^2\cos^2\frac{\theta}{2}} e^{iz(\sigma+\sigma')} \tag{A.4}$$

where the contour $C$ runs infinitesimally above and below the real axis. With $0<\sigma+\sigma'<2\pi$ we can close these paths in the upper/lower half plane, giving

$$\sum_{n\in\mathbb{Z}} \frac{n e^{in(\sigma+\sigma')}}{n^2+\widehat{m}^2\cos^2\frac{\theta}{2}} = \pi i \left( \frac{e^{-\widehat{m}\cos\frac{\theta}{2}(\sigma+\sigma')}}{1-e^{-2\pi\widehat{m}\cos\frac{\theta}{2}}} + \frac{e^{\widehat{m}\cos\frac{\theta}{2}(\sigma+\sigma')}}{1-e^{2\pi\widehat{m}\cos\frac{\theta}{2}}} \right) \tag{A.5}$$

and analogously

$$\sum_{n\in\mathbb{Z}} \frac{e^{in(\sigma+\sigma')}}{n^2+\widehat{m}^2\cos^2\frac{\theta}{2}} = \frac{\pi}{\widehat{m}\cos\frac{\theta}{2}} \left( \frac{e^{-\widehat{m}\cos\frac{\theta}{2}(\sigma+\sigma')}}{1-e^{-2\pi\widehat{m}\cos\frac{\theta}{2}}} - \frac{e^{\widehat{m}\cos\frac{\theta}{2}(\sigma+\sigma')}}{1-e^{2\pi\widehat{m}\cos\frac{\theta}{2}}} \right). \tag{A.6}$$

Plugging this into (A.1), it can be seen with (6.50) that these terms are exactly cancelled by the zero-mode contributions. Altogether this leads to

$$\{S(\tau,\sigma),S(\tau,\sigma')\} = \sum_{n\in\mathbb{Z}} e^{in(\sigma-\sigma')} = 2\pi\delta(\sigma-\sigma')\,; \qquad 0<\sigma,\sigma'<\pi\,, \tag{A.7}$$

which had to be shown. The remaining cases follow from a similar analysis.

### A.2 Bosons

For the Neumann directions the canonical conjugated momenta are given by

$$\begin{aligned} P^I &= \frac{1}{2}\left(\partial_\tau X^I + F^{IJ}\partial_\sigma X^J\right) \\ &= \frac{-i\widehat{m}}{2\sin\frac{\theta}{2}} \left( e^{-i\widehat{m}\sin\frac{\theta}{2}\tau} \exp[iJ\widehat{m}\cos\frac{\theta}{2}\sigma]^{IJ} a^J - e^{i\widehat{m}\sin\frac{\theta}{2}\tau} \exp[-iJ\widehat{m}\cos\frac{\theta}{2}\sigma]^{IJ} a^{\dagger J} \right) + \\ &\quad + \frac{1}{2} \sum_{n\in\mathbb{Z}\backslash\{0\}} e^{-i\omega_n\tau} \left( \left[1-\frac{n}{\omega_n}F\right]^{IJ} \alpha_n^J e^{in\sigma} + \left[1+\frac{n}{\omega_n}F\right]^{IJ} \tilde{\alpha}_n^J e^{-in\sigma} \right). \end{aligned} \tag{A.8}$$

Using this, (6.28) and $[\alpha_n^I, \alpha_m^J] = \omega_n \delta_{n+m}\delta^{IJ}$, the nonzero mode contributions to $[X^I(\tau,\sigma), P^J(\tau,\sigma')]$ are given by

$$i\sum_{n\in\mathbb{Z}} e^{in(\sigma-\sigma')} - i\sum_{n\in\mathbb{Z}} \frac{\widehat{m}^2\cos^2\frac{\theta}{2} - n^2}{n^2+\widehat{m}^2\cos^2\frac{\theta}{2}} e^{in(\sigma+\sigma')}\,. \tag{A.9}$$

Using the contour integration as before, this leads to

$$i\delta(\sigma-\sigma') - 2\pi i\widehat{m}\cos\frac{\theta}{2}\left[ \frac{e^{-\widehat{m}\cos\frac{\theta}{2}(\sigma+\sigma')}}{1-e^{-2\pi\widehat{m}\cos\frac{\theta}{2}}} - \frac{e^{\widehat{m}\cos\frac{\theta}{2}(\sigma+\sigma')}}{1-e^{2\pi\widehat{m}\cos\frac{\theta}{2}}} \right]. \tag{A.10}$$

Setting $[a^I, a^{J\dagger}] = L^{IJ}$, the zero mode contribution are here

$$\frac{i\widehat{m}}{4\sin\frac{\theta}{2}} \left( e^{\widehat{m}\cos\frac{\theta}{2}(\sigma+\sigma')}((1+iJ)L+(1-iJ)L^t) + e^{-\widehat{m}\cos\frac{\theta}{2}(\sigma+\sigma')}((1-iJ)L+(1+iJ)L^t) \right).$$

Comparing this with (A.10) again uniquely determines $L$ as given in (6.48).



JHEP06(2005)041

## B. The modular transformation

Using the result

$$f_2^{(m)}(q) = f_4^{(\widehat{m})}(\widetilde{q}) \tag{B.1}$$

from [14], we have to establish the relation

$$g_2^{(m)}(t,\theta) = \widehat{g}_4^{(\widehat{m})}(\widetilde{t},\theta) \tag{B.2}$$

with $\widetilde{t} = \frac{1}{t}$ for the special functions defined in (5.6) and (6.58).

Setting $m_1 = m\cos\frac{\theta}{2}$ the proof given in appendix D of [15] carries over immediately to the present situation, so that we will not reproduce it here in detail. In that proof the open string offset $\overline{\Delta}_{\widehat{m},\theta}$ is determined to

$$\overline{\Delta}_{\widehat{m},\theta} = -\frac{1}{(2\pi)^2}\sum_{p=1}^{\infty}(-1)^p \sum_{r=0}^{\infty} c_r^p\, \widehat{m} \left(\frac{\partial}{\partial \widehat{m}^2}\right)^r \frac{1}{\widehat{m}} \int_0^{\infty} ds \left(\frac{-s}{\pi^2\cos^2\frac{\theta}{2}}\right)^r e^{-p^2 s - \frac{\pi^2\widehat{m}^2}{s}}\,, \tag{B.3}$$

where the coefficients $c_r^p$ are taken from the power series expansion

$$\left(\frac{\omega_n + m_1}{\omega_n - m_1}\right)^p + \left(\frac{\omega_n - m_1}{\omega_n + m_1}\right)^p = \sum_{r=0}^{\infty} c_r^p \left(\frac{\omega_n}{m_1}\right)^r. \tag{B.4}$$

As it should, the limit $\theta \to 0$ reproduces the instanton result. The limit $\theta \to \pi$, however, is singular, as the expansion (B.4) as used in the derivation of (B.3) strictly makes sense only when understood as an analytic continuation for the meromorphic function on the left hand side of (B.4) to values

$$m_1 = m\cos\frac{\theta}{2} < \omega_n = \sqrt{n^2 + m^2}\,. \tag{B.5}$$

Doing this, only the $r = 0$ term in (B.3) contributes with a factor $c_0^p \to 2$, leading to

$$\overline{\Delta}_{\widehat{m},\theta} \to 2\Delta'_m \tag{B.6}$$

with

$$\Delta'_m = -\frac{1}{(2\pi)^2}\sum_{p=1}^{\infty}(-1)^p \int_0^{\infty} ds\; e^{-p^2 s - \frac{\pi^2 m^2}{s}} \tag{B.7}$$

in accordance with the results of [14].

As in the last step, parts of the proof of (B.2) make use of an analytic continuation in $m_1$. It would be interesting to see whether there is a more direct approach as for example found by Gannon in [44] for the $f_i^{(m)}$ functions of [14], where certain $\theta$-function identities are being used.

## References


[1] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *A new maximally supersymmetric background of IIB superstring theory*, *JHEP* **01** (2002) 047 [hep-th/0110242].

[2] R.R. Metsaev, *Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, *Nucl. Phys.* **B 625** (2002) 70 [hep-th/0112044].

[3] R.R. Metsaev and A.A. Tseytlin, *Exactly solvable model of superstring in plane wave Ramond-Ramond background*, *Phys. Rev.* **D 65** (2002) 126004 [hep-th/0202109].

[4] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and pp waves from $N = 4$ super Yang-Mills*, *JHEP* **04** (2002) 013 [hep-th/0202021].

[5] D. Sadri and M.M. Sheikh-Jabbari, *The plane-wave/super Yang-Mills duality*, *Rev. Mod. Phys.* **76** (2004) 853 [hep-th/0310119].

[6] J.M. Maldacena, *Tasi 2003 lectures on AdS/CFT*, hep-th/0309246.

[7] J.C. Plefka, *Lectures on the plane-wave string/gauge theory duality*, *Fortschr. Phys.* **52** (2004) 264 [hep-th/0307101].

[8] A. Pankiewicz, *Strings in plane wave backgrounds*, *Fortschr. Phys.* **51** (2003) 1139 [hep-th/0307027].

[9] R. Russo and A. Tanzini, *The duality between IIB string theory on pp-wave and $N = 4$ SYM: a status report*, *Class. Quant. Grav.* **21** (2004) S1265–2196 [hep-th/0401155].

[10] K. Skenderis and M. Taylor, *Branes in AdS and pp-wave spacetimes*, *JHEP* **06** (2002) 025 [hep-th/0204054].

[11] K. Skenderis and M. Taylor, *Properties of branes in curved spacetimes*, *JHEP* **02** (2004) 030 [hep-th/0311079].

[12] P. Bain, P. Meessen and M. Zamaklar, *Supergravity solutions for D-branes in Hpp-wave backgrounds*, *Class. and Quant. Grav.* **20** (2003) 913 [hep-th/0205106].

[13] M. Billo and I. Pesando, *Boundary states for gs superstrings in an Hpp wave background*, *Phys. Lett.* **B 536** (2002) 121 [hep-th/0203028].

[14] O. Bergman, M.R. Gaberdiel and M.B. Green, *D-brane interactions in type-IIB plane-wave background*, *JHEP* **03** (2003) 002 [hep-th/0205183].

[15] M.R. Gaberdiel and M.B. Green, *The d-instanton and other supersymmetric D-branes in IIB plane-wave string theory*, *Ann. Phys. (NY)* **307** (2003) 147 [hep-th/0211122].

[16] M.R. Gaberdiel, M.B. Green, S. Schafer-Nameki and A. Sinha, *Oblique and curved D-branes in IIB plane-wave string theory*, *JHEP* **10** (2003) 052 [hep-th/0306056].

[17] A. Dabholkar and S. Parvizi, *Dp-branes in pp-wave background*, *Nucl. Phys.* **B 641** (2002) 223 [hep-th/0203231].

[18] K. Skenderis and M. Taylor, *Open strings in the plane wave background, I: quantization and symmetries*, *Nucl. Phys.* **B 665** (2003) 3 [hep-th/0211011].

[19] K. Skenderis and M. Taylor, *Open strings in the plane wave background, II: superalgebras and spectra*, *JHEP* **07** (2003) 006 [hep-th/0212184].

[20] P. Bain, K. Peeters and M. Zamaklar, *D-branes in a plane wave from covariant open strings*, *Phys. Rev.* **D 67** (2003) 066001 [hep-th/0208038].

[21] C.-S. Chu and P.-M. Ho, *Noncommutative D-brane and open string in pp-wave background with b-field*, *Nucl. Phys.* **B 636** (2002) 141 [hep-th/0203186].


JHEP06(2005)041

[22] L.F. Alday and M. Cirafici, *An example of localized D-branes solution on pp-wave backgrounds*, *JHEP* **05** (2003) 006 [hep-th/0301253].

[23] D. Sadri and M.M. Sheikh-Jabbari, *String theory on parallelizable pp-waves*, *JHEP* **06** (2003) 005 [hep-th/0304169].

[24] M. Alishahiha, M.A. Ganjali, A. Ghodsi and S. Parvizi, *On type-IIA string theory on the pp-wave background*, *Nucl. Phys.* **B 661** (2003) 174 [hep-th/0207037].

[25] S.-j. Hyun, J. Park and H.-j. Shin, *Covariant description of D-branes in IIA plane-wave background*, *Phys. Lett.* **B 559** (2003) 80 [hep-th/0212343].

[26] Y. Kim and J. Park, *Boundary states in IIA plane-wave background*, *Phys. Lett.* **B 572** (2003) 81 [hep-th/0306282].

[27] J. Polchinski and Y. Cai, *Consistency of open superstring theories*, *Nucl. Phys.* **B 296** (1988) 91.

[28] J. Lucietti, S. Schafer-Nameki and A. Sinha, *On the exact open-closed vertex in plane-wave light-cone string field theory*, *Phys. Rev.* **D 69** (2004) 086005 [hep-th/0311231].

[29] J. Lucietti, S. Schafer-Nameki and A. Sinha, *On the plane-wave cubic vertex*, *Phys. Rev.* **D 70** (2004) 026005 [hep-th/0402185].

[30] K.-S. Cha, B.-H. Lee and H.S. Yang, *A complete classification of D-branes in type-IIB plane wave background*, *JHEP* **03** (2004) 058 [hep-th/0310177].

[31] J. Maldacena and L. Maoz, *Strings on pp-waves and massive two dimensional field theories*, *JHEP* **12** (2002) 046 [hep-th/0207284].

[32] Y. Hikida and S. Yamaguchi, *D-branes in pp-waves and massive theories on worldsheet with boundary*, *JHEP* **01** (2003) 072 [hep-th/0210262].

[33] D. Kamani, *Noncommutativity and its freedoms due to the pp-wave and background gauge fields*, *Mod. Phys. Lett.* **A 18** (2003) 2125 [hep-th/0308017].

[34] R.R. Metsaev, *Supersymmetric D3 brane and $N = 4$ SYM actions in plane wave backgrounds*, *Nucl. Phys.* **B 655** (2003) 3 [hep-th/0211178].

[35] S.S. Pal, *Solution to worldvolume action of D3 brane in pp-wave background*, *Mod. Phys. Lett.* **A 17** (2002) 1735 [hep-th/0205303].

[36] G. D'Appollonio and E. Kiritsis, *D-branes and BCFT in Hpp-wave backgrounds*, *Nucl. Phys.* **B 712** (2005) 433 [hep-th/0410269].

[37] A. Abouelsaood, C.G. Callan Jr., C.R. Nappi and S.A. Yost, *Open strings in background gauge fields*, *Nucl. Phys.* **B 280** (1987) 599.

[38] M.B. Green and M. Gutperle, *Light-cone supersymmetry and D-branes*, *Nucl. Phys.* **B 476** (1996) 484 [hep-th/9604091].

[39] M.B. Green and M. Gutperle, *Comments on three-branes*, *Phys. Lett.* **B 377** (1996) 28 [hep-th/9602077].

[40] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *JHEP* **09** (1999) 032 [hep-th/9908142].

[41] S. Ghoshal and A.B. Zamolodchikov, *Boundary s matrix and boundary state in two-dimensional integrable quantum field theory*, *Int. J. Mod. Phys.* **A 9** (1994) 3841 [hep-th/9306002].

[42] H. Takayanagi and T. Takayanagi, *Notes on giant gravitons on pp-waves*, *JHEP* **12** (2002) 018 [hep-th/0209160].

[43] C.-S. Chu and P.-M. Ho, *Noncommutative open string and D-brane*, *Nucl. Phys.* **B 550** (1999) 151 [hep-th/9812219].

[44] T. Gannon, *Variations on Dedekind's eta*, in *CRM Proceedings & Lecture Notes* **34**, 2004.